On Designing of a High Voltage Standard Capacitor Using a Semi-Analytical Field Computation Method

N. K. Kishore
Professor, Dept. of Electrical Engineering
Indian Institute of Technology Kharagpur
Kharagpur, West Bengal, INDIA
kishor@ee.iitkgp.ernet.in

Gururaj S. Punekar
Associate Professor, Dept. of E & E
National Institute of Technology
Karnataka, Mangalore, INDIA
gsp@nitk.ac.in

Abstract—Charge Simulation Method (CSM) is a semi-analytical method. It is an integral equation technique of computing electric fields. Using the method, a High Voltage (HV) standard capacitor is designed. The design details are discussed along with the potential electric stress distribution in the HV-standard capacitor. The method makes use of the inherent advantages of CSM (being semi-analytical method) of getting the Capacitance value in terms of fictitious charges and the potential.

I. INTRODUCTION

Practical measurements often involve measuring the capacitance and dissipation factor of insulation concurrently [1]. This requires high voltage standard capacitor which is lossless and corona free. The simplest configuration is using coaxial cylinder arrangement. High voltage coaxial capacitors have re-entrant edges to make them corona-free. Design of such a capacitor involves the analysis of stresses at the center of the capacitor and also at the edges. Since there is no analytical solution for such a configuration, numerical techniques such as Finite Element Method (FEM), Finite Difference Method (FDM), Monte Carlo Method (MCM), and Charge Simulation Method (CSM) can be used to model it [2-3]. The CSM due to its favorable characteristics is very commonly used for field analysis of HV insulation systems. For simple physical systems, it is usually possible to find an analytical solution to evaluate capacitance (concentric spheres, co-axial cylinder with infinite length etc) [4]. However, in many cases, the physical systems are so complex that it is extremely difficult, if not impossible, to find analytical solutions. The analytical expression [4] derived for the capacitance of a coaxial cable per unit length is used for the approximate design of the coaxial capacitor. The dimensions thus obtained serve as an initial estimate for the CSM Model.

The Coaxial capacitor with air as the insulating medium is simulated using Ring charges. The design values of maximum stress and capacitance are verified using the CSM Model. The actual capacitor is fabricated using the calculated dimensions and is
tested for the designed peak voltage. The fabricated capacitor which is corona-free can be used as a standard capacitor in Dissipation Factor measurements. Standard capacitors can also be used in measurement of peak value of alternating voltages.

This paper mainly discusses the application of CSM in designing a 100 pF, 12 kV, HV standard capacitor. A standard capacitance of 100 pF which is useful in High Voltage applications is designed using Charge Simulation Method (CSM). An air insulated (dielectric) capacitor which can be used up to 12 kV is designed which is corona free. The capacitor configuration with re-entrant edges makes the capacitor corona free, thus makes it suitable for use as a standard capacitor in High Voltage applications. CSM using uniform ring charges is implemented for simulating the capacitor. The inherent advantage of using CSM for such a purpose is discussed.

II. SIMULATION

A. CSM Model details

The analysis of distribution of the electric field in a standard capacitor is very important for proper design making it corona free, particularly in high voltage applications. The stress in a coaxial air capacitor depends on the geometrical configuration as well as on the absolute dimensions and the applied voltage [4-6]. In the present study a coaxial cylinder, air-filled (atmospheric air) capacitor is simulated using the ring charges in developing CSM model. Calculations are repeated using a digital computer changing one or several of the following parameters:

(a) Number of simulation charges.
(b) Location of simulation charges.
(c) Location of contour points.

The calculations are repeated by varying the above parameters until the simulation errors are within acceptable limits [3]. The CSM model is verified by comparing the radial potential and electric field intensity magnitudes computed with those of the analytical solution available for a coaxial cable of infinite length (in its mid regions, where end effects can be neglected). Having thus validated the CSM model (figure 1), it is then used to design the coaxial capacitor of desired capacitance by varying its dimensions. The variation of capacitance of the simulated capacitor with its dimensions is discussed in section IV.

B. About CSM

In the present simulation, the charges associated with inner cylinder of radius r is replaced by N_{ci} number of ring charges having a radius of r_{ci}; and the charges associated with outer cylinder of radius R is replaced by N_{co} number of ring charges of radius R_{co}; where r_{ci} > r and R_{co} > R. In order to determine the magnitude of these charges, N_p test points (N_p=N_{ci}+N_{co}) on the surface of the inner and outer cylinders with known potential of 1 and 0 p.u. (per unit), respectively, are chosen. These test points, their locations in relation with those of fictitious charges can be used to form N equations with N charge magnitudes as unknowns. These equations have to satisfy the boundary conditions (the potential of the points on the inner & outer cylindrical surfaces). This can be represented
in the matrix form as given by the equation (1) [3].

\[
[V] = [P] \times [Q]
\]  

(1)

Where

- \([P]\) is the potential coefficient matrix.
- \([V]\) is the vector containing potential values of boundary points (contour points).
- \([Q]\) is the vector of simulating charges.

Each element of matrix \([P]\) depends on the type of the charge and the hence its coefficient [3]. In the present study the ring charges and their coefficients are used for simulating the required geometry with potential and electric fields (Figure 1).

The magnitude and position of ring charges is determined to satisfy the boundary by appropriately choosing the number of charges, number of contour points and their locations. Thus obtained set of charges should satisfy the boundary conditions of the potential.
The error in simulation using CSM can be estimated by choosing a large number of test points on the boundary of the electrodes to compute potential $V$ and electric field intensity $E$ at each of these points using the equations (1) and (2), respectively:

$$[E] = [F]^* [Q] \quad (2)$$

Where $[E]$ is the field coefficient matrix.

The calculated value of potential and electric field intensity at the check points satisfy the boundary conditions set at the inner electrode (1 V) and at the outer electrode (0 V). Also the radial potential and electric field intensity measured along the midpoint of the capacitor matches with those of the analytical solution (derived for the coaxial cable of infinite length), thus validating the model.

C. Standard capacitor and its design

Laboratory standard capacitors must be stable and reproducible in order to be considered as standards [7-9]. Capacitance is measured by balancing the current through the capacitor under test against the current through a standard air or compressed gas capacitor using some type of bridge arrangement [1-2]. There are a variety of such bridges described in literature [1]. The one most commonly used in high voltage applications for the last sixty years is the Schering Bridge [2]. The series capacitance method [2] is a simple but accurate method for measurement of peak values of a.c voltages. This method can also be used to measure the capacitance of a High Voltage capacitor if the peak value of the a.c input is known.

The 12 kV, 100 pF coaxial air capacitor consisting of two steel electrodes placed coaxially with air as the dielectric medium between them is designed. The design process to arrive at various physical dimensions was started with the approximate estimations using coaxial cable arrangement of infinite length. These dimensions are then fine-tuned by CSM model developed, by numerical experimentations. The radius and height of the inner electrode is 10.35 cm & 88.3 cm, respectively. The radius and height of the outer electrode is 14.8 cm & 50.3 cm respectively. The outer electrode is placed on a plexi glass support (24 cm in height) and the inner electrode is placed on a PVC support (5 cm in height). The distance of the air-gap between the electrodes is 4.47 cm. The capacitance value of the model implemented in CSM is checked. (This capacitor designed has been fabricated and tested experimentally; details of experimental tests are not given in this paper, but will be presented during the conference; the simulated value of capacitance matches with the designed value within ±3%).

III. CALCULATION OF CAPACITANCE

A. Approximate estimate assuming coaxial arrangement

The capacitance of a coaxial capacitor without re-entrant edges for a length of ‘ L ‘ m is given by equation (3), using the analytical solution for an infinitely long coaxial cylinder arrangement [2,4].
The electric stress at the inner electrode is limited to 5 kV/cm (peak) and is given by equation (4), using the analytical solution for an infinitely long coaxial cylinder arrangement [2,4].

\[
E = \frac{V_{\text{applied}}}{r \times \ln\left(\frac{R}{r}\right)}
\]  

(4)

where
\(V_{\text{applied}}\) = Potential of the inner electrode (V).
\(L\) = Length of the capacitor (m).
\(R\) = Radius of the outer electrode (m).
\(r\) = Radius of the inner electrode (m).
\(\varepsilon_o = 8.854 \times 10^{-12}\) (F/m) is the permittivity of the air (or free space).

For \(r=10\) cm, \(R/r = 1.41\) and \(V_{\text{sys}} = 12\) kV, the following design values are obtained using equations (3) and (4) given above:

\(R = r \times 1.41 = 14.1\) cm.
\(d = R - r = 4.1\) cm.
\(E_{\text{max}} = 4.93\) kV/cm.

For a capacitance of 100 pF and substituting the above values of \(R\) and \(r\) in eqn. (7), we obtain
\(L = 61\) cm and the corresponding \(C_{\text{design}} = 98.76\) pF.

The above procedure is based on the analytical expressions available [2,4] for the coaxial cable of infinite length and thus gives an approximate result for capacitance & Electric field strength. The results obtained are used as initial estimates for simulation and fabrication.

### B. Estimations using CSM

The CSM code is developed, which is specific to the model described in figure 1. Ring charges are placed as shown in that figure outside the outer (low voltage electrode) conductor and inside the inner (high voltage electrode) conductor. Their (ring charges) magnitudes are computed using the equations (1). The infinite ground plane is simulated using the ground image electrodes and image charges as shown in the figure 1.

Using the CSM model developed a number of numerical experiments are conducted by varying the length of the electrodes. Figure 2 shows the variation of capacitance with the length of the capacitor with re-entrant edge radius equal to 10 cm. This is compared with the variation in capacitance with length given by analytical expression (equation 3). It can be observed from the plot that the value of capacitance obtained by simulation is greater...
than that obtained by the analytically designed value of 100 pF (for L=0.6 m). This is because of the additional capacitance between the Re-entrant edges and the ground plane which gets added to the designed value of capacitance. Since the analytical expression is derived for a coaxial capacitor without re-entrant edges, the value of capacitance obtained for the actual configuration via CSM is more accurate.

In order to confirm the effect of re-entrant edges on the capacitance magnitude the CSM model with re-entrant edges are compared with the analytical values. Figure 3 shows the variation of capacitance with length for such a situation. The simulated values of capacitances almost match with the analytical values since the re-entrant edge radii are now set to zero in simulation. Here the simulated value of capacitance is approximately equal to 100 pF for a length L=0.6 m.

Fig. 2. Plot of capacitance versus length of the capacitor taking into account the considering re-entrant edges.

![Capacitance vs Length](image)

Fig. 3. The variation of capacitance with length. The simulated value of capacitance almost matches with the analytical values since the re-entrant edge radii are set to zero in this simulation.
IV. NUMERICAL EXPERIMENTS

The capacitor model (CSM model) is used to study the variation of stress distribution with change in the re-entrant edge radius ($r_e$; in figure 1), the clearance gap distance above ground ($x_2$; in figure 1) and the length ($L$; in figure 1) of the capacitor.

The electric field and potential variation at the center (where there will be no or negligible edge effects) of the capacitor model obtained by simulation using ring charges (figures 4 & 5). The CSM model results match well with the analytical solution derived for the coaxial capacitor without re-entrant edges. It is observed that the stress at the center of the capacitor will be less than that at the edges.

Fig. 4. Plot of radial potential with inner (high voltage) electrode at 1 V (p.u.) and outer (low voltage) electrode at 0 V (at the center of the cylindrical coaxial arrangement in the air gap).

Fig. 5. Plot of radial field with inner (high voltage) electrode at 1 V (p.u.) and outer (low voltage) electrode at 0 V (at the center of the cylindrical coaxial arrangement in the air gap).
The electric stress at the upper edge is greater compared to that at the lower edge (nearer to the ground plane). The variation of stress on the inner surface of the outer cylinder for different values of re-entrant edge radius is obtained (figure 6 (a)-(c)) by conducting the numerical experiments on the CSM model developed. It is observed that the stress at the edges rises as the value of the re-entrant edge radius is decreased (from 10 cm. to 8.5 cm.).

![Electric Stress Graph](image)

**Fig. 6.** Plot of surface electric field on the outer electrode with inner electrode at 1 V and outer electrode at 0 V (for re-entrant edge radius of the outer electrode equal to (a) 10 cm (b) 9 cm (c) 8.5 cm.

V. **CONCLUSION**

The CSM model based design of standard capacitors helps in analysing the electric stresses at various points in the capacitor. This helps in designing a corona free (partial discharge free) capacitor. The CSM being an integral equation technique, calculates the fictitious charge magnitudes, which in actuality will be equal to the magnitude of the actual charge magnitude residing on the electrodes. This helps in estimating the capacitance (C) directly; as the potential (V) and charge magnitudes (Q) are known (C=Q/V).

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REFERENCES


